

A semi-analytical model of disk evaporation by thermal conduction

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Abstract. The conditions for disk evaporation by electron thermal conduction are examined, using a simplified semi-analytical 1-D model. The model is based on the mechanism proposed by Meyer & Meyer–Hofmeister (1994) in which an advection dominated accretion flow evaporates the top layers from the underlying disk by thermal conduction. The evaporation rate is calculated as a function of the density of the advective flow, and an analysis is made of the time scales and length scales of the dynamics of the advective flow. It is shown that evaporation can only completely destroy the disk if the conductive length scale is of the order of the radius. This implies that radial conduction is an essential factor in the evaporation process. The heat required for evaporation is in fact produced at small radii and transported radially towards the evaporation region.

Key words: hydrodynamics – black hole physics – accretion, accretion disks

1. Introduction

Ever since their theoretical rediscovery, advection dominated accretion flows (ADAFs, Abramowicz et al. 1995; Narayan & Yi 1994, 1995; Ichimaru 1977) have been widely regarded as the most likely source of Comptonized X-ray radiation observed from many X-ray binaries. On the basis of observational evidence (Lasota 1996, Narayan et al. 1997, Hameury et al. 1997 and references therein) it is believed that ADAFs form the inner part of the accretion disk system, while the outer part is formed by a standard Shakura–Sunyaev disk (SSD, Shakura & Sunyaev 1973). Despite the fact that such a bimodal disk geometry has already been proposed a long time ago (Thorne & Price, 1975; Shapiro, et al. 1976, henceforth SLE), no satisfactory theoretical explanation for these disk transitions has so far been found.

It seems plausible that disk surface evaporation is responsible for the transition. By some mechanism, the uppermost layers of the disk are heated up faster than radiative cooling can cool them down. The resulting hot ‘vapor’ forms a corona on top of the disk. Part of this vapor then accretes towards the central object, while another part moves outwards via a transsonic wind or breeze (Meyer & Meyer–Hofmeister 1994, Liu et al. 1997, Dullemond & Turolla 1998). At a certain radius R_{evap} the entire

disk has been evaporated, and the ‘corona’ therefore becomes a true ADAF within this evaporation radius.

It remains uncertain what drives the surface evaporation. Several mechanisms have been proposed. When no corona exists beforehand, one can show that the upper layers of the SSD are unstable with respect to thermal perturbations (Shaviv & Wehrse 1986; Hubeny 1989; Tschäpe & Kley 1993). These layers will heat up in a run-away fashion, thereby causing the evaporation of the upper layers of the disk. Also, acoustic effects (Icke 1976) and magneto-hydrodynamical fluctuations and instabilities may produce a corona, in a way similar to the formation of the solar corona (Galeev et al. 1979; Tout & Pringle 1992).

The most promising mechanism for disk evaporation is electron thermal conduction (Meyer & Meyer–Hofmeister 1994, henceforth MMH). A pre-existing hot ‘corona’ injects energy into an extremely thin layer on top of the underlying disk, turning this material into new hot coronal plasma. This mechanism is studied in this paper.

In their paper, Meyer & Meyer–Hofmeister treat the problem in a 1-D vertical way, and obtain semi self-consistent models of disk evaporation. The goal of this paper is to re-examine this evaporation mechanism, but without the aim of finding fully consistent models. In fact, due to the complexity of the gas dynamics in the evaporation region, it is questionable whether it is possible to model this process in a consistent way by use of 1-D methods (Abramowicz et al. 1997). Instead, the goal of this paper is to study the structure of the boundary layer connecting the ADAF to the disk, and to study in a qualitative way the mechanism of evaporation, the conditions under which evaporation is possible, and the extent of validity of a 1-D vertical treatment of this evaporation process. To this end, a second order ordinary differential equation (ODE), containing only the most essential ingredients for the description of the evaporation process, is derived. This equation is subsequently reduced to a dimensionless form containing two dimensionless constants. The resulting dimensionless equation is then solved and a simple relation between the two dimensionless constants is obtained by requiring a self-consistency condition to be fulfilled. Finally, by transforming the dimensionless solutions back to the real problem of disk evaporation, several important consequences are derived.

2. Description of the model

In order to explore the basic physics of evaporation by thermal conduction, we study the simplest possible situation: a hot plasma flow on top of an accretion disk, with vertical thermal conduction, viscous heating, advective cooling and radiative cooling. This simplified conception of the problem should suffice for the goal of this paper. In principle one could think of this problem as the problem of an ordinary ADAF and an SSD co-existing at each radius, and vertically glued together by a boundary layer. Such a model is consistent as long as the boundary layer is thin and the evaporation is not too strong (Dullemond & Turolla 1998). Under these circumstances one can model the ADAF locally as a self-similar ADAF (Narayan & Yi 1994). It is expected that the qualitative conclusions of the present model also remain valid somewhat beyond the breakdown of self-similarity.

In the boundary layer the temperature drops steeply from the ADAF temperature down to the chromospheric disk temperature. The vertical scale associated with this gradient is extremely small, much smaller than the disk thickness. This justifies the use of a 1-D method for at least the lower parts of the boundary layer. The base of the boundary layer lies above the chromosphere of the disk, so that an optically thin treatment of the boundary layer is sufficient. The temperature gradient constitutes a heat flux pointing downwards towards the surface of the disk. At relatively high altitudes the ADAF produces an excess of energy which is transported downwards by the heat flux. In the lower parts of the boundary layer both radiative cooling and the upward gas motion absorb the flux. The upward gas motion, acts as a kind of vertical advective cooling (MMH). It is precisely large enough to cancel the downward flux, guaranteeing the flux at the base to be zero. This condition of zero flux at the base determines the evaporation rate.

As the matter moves upwards from the disk surface, it has the tendency to shift towards larger radii. This is because, as the temperature increases, the pressure support against gravity increases. A new gravitational force balance can only be reached by moving radially outwards a bit. But this phenomenon has no significant effect on the energy balance of the boundary layer, and we will ignore it. Also we will ignore the vertical friction ($t^{z\phi}$). We do take into account the decrease of Ω as matter moves upwards into the corona/ADAF.

The viscosity prescription is a delicate matter in boundary layers of the type studied here. Friction is assumed to arise from magneto-turbulence. The length scale associated with this turbulence is usually chosen ad-hoc, using the famous alpha-viscosity prescription: $l = \alpha H$. For the disk height H one usually takes the rough estimate $H = c_s/\Omega_K$, following from vertical pressure balance. However, the length scales of magneto turbulence in the boundary layer are uncertain. If one would take $l = \alpha c_s/\Omega_K$ then one finds that this length scale exceeds the boundary layer thickness, $l \gg z$, close to the lower boundary. This is inconsistent. In order to avoid having to discuss this highly uncertain issue, we rather ignore the effects of vertical friction. In principle the radial friction $t^{r\phi}$ suffers from the

same disease, but its contribution to the energy equation becomes small in the lower parts of the boundary layer, so this effect can be ignored here as well.

3. The equations

As a lower boundary to the calculational domain I take the height where the coronal temperature becomes equal to the chromospheric temperature. The height coordinate z is gauged to zero at this lower boundary, so that $z = 0$ represents the height above the chromosphere. As an upper boundary to the calculational domain I take a height z_{up} obeying roughly $z_{up} \lesssim 0.3R$ (R being the radius at which the corona is studied), so that geometric effects of the flow can safely be ignored. The corona extends well above this upper boundary, by virtue of the fact that it is assumed to be an advection dominated flow, for which the thickness H is roughly equal to the radius R .

The model describes the temperature T , the density ρ and the vertical velocity v as a function of z . I presume that the system is quasi-stationary. The equations for T , ρ and v are the compressible Navier-Stokes equations in the coordinates R (radius), z (height) and ϕ (azimuth), but in this calculation I choose the corona to be axisymmetric, and self-similar in the radial direction, thus reducing the problem to a one-dimensional problem in the coordinate z .

The continuity equation is $(\rho v)' = 0$ which integrates to

$$\rho v = \frac{1}{2} \Psi \quad (1)$$

The radial motion and geometrical terms are neglected. It is assumed that the radial accretion predominantly takes place at altitude $z \gtrsim z_{up}$, allowing us to regard the evaporation rate as a constant of motion in the domain of interest. The integration constant Ψ is the evaporation rate in units $\text{g cm}^{-2}\text{s}^{-1}$. The factor $1/2$ accounts for the fact that the disk has two sides. The pressure balance is $(\rho c_s^2)' = 0$, where c_s is the isothermal sound speed. This trivially integrates to

$$\rho c_s^2 = \Pi \quad (2)$$

The ram pressure is neglected because the motion is assumed to be very subsonic. And as in the continuity equation, the geometric terms are neglected here as well.

The energy equation consists of five terms: viscous heating Q_+ , radial advective cooling Q_{adv} , optically thin radiative cooling Q_- , electro-conductive heat flux J_c and vertical advective heat flux J_v . Viscous dissipation due to the vertical gradient of the rotational frequency, $d\Omega(z)/dz$, is neglected.

The dissipation due to the radial gradient in Ω is given by

$$Q_+ = \rho \nu \left(R \frac{d\Omega}{dR} \right)^2 \quad (3)$$

where the kinematic viscosity ν is given by

$$\nu = \frac{2}{3} \alpha \frac{c_s^2}{\Omega_K} \quad (4)$$

By using Eq. (2) the Q_+ can be written as

$$Q_+ = \frac{3}{2} \alpha \omega^2 \Omega_K \Pi \quad (5)$$

where the dimensionless rotational frequency ω is defined by $\Omega = \omega \Omega_K$, with $\omega < 1$. For an ADAF, the radial force balance equation, with omission of the radial kinetic term (which is very small), is

$$\Omega^2 R^2 - \frac{d \log(p)}{d \log(R)} c_s^2 = \Omega_K^2 R^2 \quad (6)$$

We take $p \propto R^{-5/2}$, so that this equation reduces to

$$\omega^2 = 1 - \frac{5}{2} \frac{c_s^2}{\Omega_K^2 R^2} \quad (7)$$

This can be substituted in Eq. (5), for use in the energy Eq. (17) below. The radial advective cooling is given by

$$Q_{adv} = \rho v_R \left(\frac{de}{dR} + p \frac{d\rho^{-1}}{dR} \right) \quad (8)$$

where $e = c_s^2/(\gamma - 1)$ is the thermal energy of the gas. The radial gas velocity is something that has to be estimated from the presumed radial structure of the corona. Take v_R to be the usual expression

$$v_R = -\alpha \frac{c_s^2}{\Omega_K R} \quad (9)$$

If one wants to take into account the outwards shift resulting from the requirement of pressure balance (discussed above), then one should add the velocity $v_R^{(1)}$ defined as

$$v_R^{(1)} = \frac{5}{2} \frac{R}{\omega^2} \frac{v}{\Omega_K^2 R^2} \frac{dc_s^2}{dz} \quad (10)$$

where v is again the vertical velocity. This extra velocity adds a contribution to the Q_{adv} , which is small enough not to influence the result significantly. For the sake of clarity it is ignored here, although it is easy to incorporate it. By using Eqs. (2, 9), Eq. (8) becomes

$$Q_{adv} = \alpha \left(\frac{1}{\gamma - 1} - \frac{3}{2} \right) \frac{\Pi}{\Omega_K R^2} c_s^2 \quad (11)$$

Radiative cooling is denoted with the symbol Q_- . Several cooling mechanisms can play a role, but the most important cooling mechanism in the boundary layer is Bremsstrahlung, $Q_- \simeq 5.0 \times 10^{20} \rho^2 \sqrt{T}$. The temperature T is related to c_s^2 by $kT = \mu m_p c_s^2$, where μ is the molecular weight, and m_p is the proton mass. The mean particle weights are $\mu = 0.59$, $\mu_i = 1.23$ and $\mu_e = 1.14$. The Bremsstrahlung cooling function is (using Eq. (2))

$$Q_- = K_1 \Pi^2 c_s^{-3} \quad (12)$$

The constant K_1 is defined as

$$K_1 \simeq 4.2 \times 10^{16} \text{ cm}^6 \text{ erg}^{-1} \text{ s}^{-4} \quad (13)$$

The vertical advective flux is J_v , is

$$J_v = \rho v h = \frac{1}{2} \frac{\gamma}{\gamma - 1} \Psi c_s^2 \quad (14)$$

where h is the enthalpy, and Eq. (1) has been used. The kinetic energy term has been neglected. Finally the conductive flux $J_c \simeq -9.2 \times 10^{-7} \eta T^{5/2} dT/dz$ (Braginskii 1963, Spitzer 1962) can be written in the form,

$$J_c = -K_0 c_s^5 \frac{dc_s^2}{dz} \quad (15)$$

The conductivity coefficient K_0 , for conduction along the magnetic field, is

$$K_0 \simeq 2.8 \times 10^{-35} \eta \text{ erg s}^6 \text{ cm}^{-8} \quad (16)$$

The Coulomb logarithm is roughly $\ln \Lambda \simeq 20$. The coefficient $\eta \leq 1$ is put in as a fudge factor to parameterize the reduction in mean free path length as a result of possible collective plasma modes and confinement by random magnetic fields. The microphysics of plasmas in these conditions is insufficiently understood to allow an estimate of η from first principles, so we must retain it as the main unknown parameter of the model.

The energy equation is now

$$\frac{dJ_c}{dz} + \frac{dJ_v}{dz} = Q_+ - Q_{adv} - Q_- \quad (17)$$

This is the basic equation of this paper. The density ρ and the velocity v have all been eliminated in favor of c_s , and the equation has reduced to a single second order diffusion equation.

The relation between the coronal accretion rate $\dot{M}_c \equiv -4\pi R H \rho v_R$ and the value Π is an integral of the model over the entire vertical height H . But within the range of validity of this model, a good estimate is given by assuming that the corona can be approximated as a homogeneous flow of height $H = \sqrt{5/2} \langle c_s \rangle / \Omega_K$ (where $\langle c_s \rangle$ is the average temperature in the corona). One obtains

$$\Pi \simeq \frac{\dot{M}_c}{2\pi \sqrt{10}} \frac{\Omega_K}{\alpha R \sigma} \quad (18)$$

here the symbol σ is defined as $\sigma = \langle c_s \rangle / \Omega_K R$. It follows from the solution of Eq. (28) below. For $\gamma = 1.5$ and low enough \dot{M}_c this value is $\sigma \simeq 0.59$.

4. Dimensionless form

By defining the variable y as

$$y = \xi^{7/2} \left(\frac{c_s}{\Omega_K R} \right)^7 \quad (19)$$

where ξ is

$$\xi = \frac{2}{3(\gamma - 1)} + \frac{3}{2} \quad (20)$$

and a new coordinate x as

$$x = \sqrt{\frac{21\alpha}{4K_0}} \xi^{7/4} \Pi^{1/2} \Omega_K^{-3} R^{-7/2} z \quad (21)$$

Eq. (17) becomes

$$-\frac{d^2y}{dx^2} + Dy^{-5/7}\frac{dy}{dx} = 1 - y^{2/7} - Cy^{-3/7} \quad (22)$$

This is the dimensionless form of the main equation of this paper. From left to right one has the terms representing the divergence of the conductive flux, the vertical advective cooling, the viscous heating, the radial advective cooling and finally the radiative cooling. The symbols C and D are defined as

$$C = \frac{2K_1\xi^{3/2}}{3\alpha} \frac{\Pi}{\Omega_K^4 R^3} \quad (23)$$

$$D = \frac{\gamma}{\gamma-1} \sqrt{\frac{\xi^{3/2}}{21\alpha K_0}} \frac{\Psi}{\sqrt{\Pi}} \frac{1}{\Omega_K^2 R^{3/2}} \quad (24)$$

Using Eq. (18) the constant C can be directly related to the coronal accretion rate \dot{M}_c by,

$$C = \frac{K_1\xi^{3/2}}{3\pi\sqrt{10}\alpha^2\sigma} \frac{\dot{M}_c}{\Omega_K^3 R^4} \quad (25)$$

The values of Π and Ψ are,

$$\Pi = \frac{3\alpha}{2K_1\xi^{3/2}} \Omega_K^4 R^3 C \quad (26)$$

$$\Psi = \frac{\gamma-1}{\gamma} \sqrt{\frac{63}{2}} \sqrt{\frac{K_0}{K_1}} \frac{\alpha}{\xi^{3/2}} \Omega_K^4 R^3 \sqrt{C} D \quad (27)$$

The solution for the corona at high altitude ($x \gg 1$) must be such that both fluxes J_c and J_v vanish. The equation is then simply a balance between viscous heating and advective and radiative cooling,

$$1 - y^{2/7} - Cy^{-3/7} = 0. \quad (28)$$

This equation is of rank 5 and cannot be solved exactly, but an very good parabolic approximation (to a few%) is given by

$$y = \left(\frac{1}{2} \pm \frac{1}{2} \sqrt{1 - \frac{125}{6\sqrt{15}}C} \right)^{28/11} \quad (29)$$

For $C \leq C_{crit} \equiv 6\sqrt{15}/125$ this equation has two branches of solutions. The upper branch represents the ADAF branch y_{adaf} , while the lower branch represents the SLE branch y_{sle} (SLE 1976). For most of the solutions discussed in this paper, the corona is found in the ADAF state, $y(\infty) = y_{adaf}$. At $C = C_{crit} \equiv 6\sqrt{15}/125$ the two branches meet. This value C_{crit} represents the critical accretion rate for optically thin accretion flows (Abramowicz et al. 1995),

$$\dot{M}_{crit} = \frac{18\pi\sqrt{6}}{25} \frac{\alpha^2\sigma}{K_1\xi^{3/2}} \Omega_K^3 R^4 \quad (30)$$

The \dot{M}_c and C relate to each other as $\dot{M}_c/\dot{M}_{crit} = C/C_{crit}$.

5. Solution without evaporation

As a simple illustration let's first solve the equations without evaporation, so simply put D to zero. Eq. (22) becomes

$$-\frac{d^2y}{dx^2} = 1 - y^{2/7} - Cy^{-3/7} \quad (31)$$

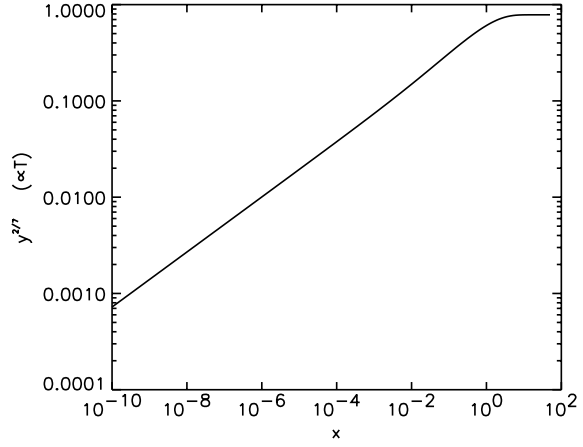


Fig. 1. The temperature structure in dimensionless variables. Note that $y^{2/7}$ is proportional to the temperature. Both axes are logarithmic to clarify the structure. This solution is for $D = 0$ and $C = 0.15$.

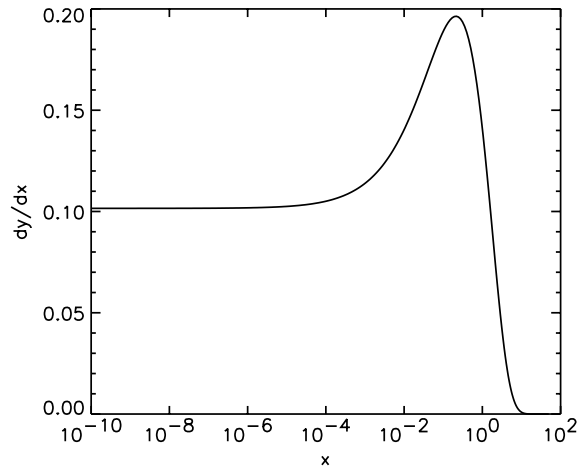


Fig. 2. The dimensionless conductive heat flux, plotted linearly. The solution is the same as for Fig. 1, for $D = 0$ and $C = 0.15$.

At the upper boundary $x = x_{up}$ the thermal conductive flux should vanishes: $J_c \propto dy/dx = 0$. At the lower boundary $x = 0$ the temperature should vanish, $y = 0$. The latter condition is of course unphysical, since the temperature should equal the chromospheric temperature rather than zero. But since the chromospheric temperature is presumed to be very low compared to the coronal temperature, this approximation is very good (within 1%).

By taking the first integral of Eq. (31), one finds the following expression for the dimensionless heat flux $j_c \equiv dy/dx \propto J_c$,

$$j_c^2 \equiv \left(\frac{dy}{dx} \right)^2 = -2y + \frac{14}{9}y^{9/7} + \frac{7}{2}Cy^{4/7} + 2y_\infty - \frac{14}{9}y_\infty^{9/7} - \frac{7}{2}Cy_\infty^{4/7}$$

where y_∞ is the value of $y(x \rightarrow \infty)$, which is a solution to Eq. (28). This equation can be integrated numerically (using a relaxation scheme, for example), and the results are plotted in Figs. 1 and 2. The result shows that at $x = x_{up}$ the corona is in

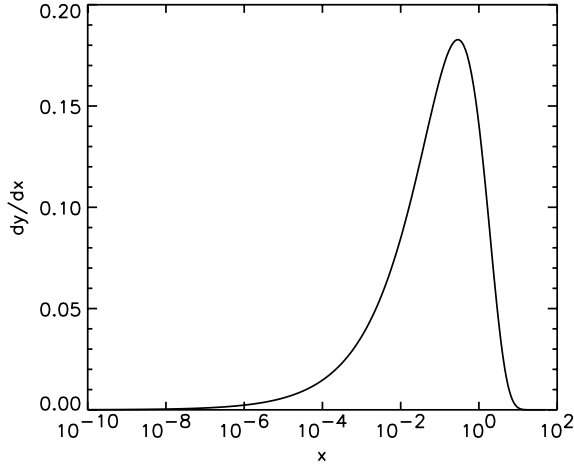


Fig. 3. The dimensionless conductive heat flux, plotted linearly. By requesting the flux at the base to vanish one finds $D = 0.2055$ for $C = 0.15$.

the ADAF state ($y = y_{adaf}$). Closer to the base of the corona the temperature drops and the conductive heat flux starts to grow. It has a maximum when $y(x) = y_{sle}$, at a certain $x = x_{sle}$. For $x > x_{sle}$ there is excess heating and flux is being produced. For $x < x_{sle}$ there is excess cooling and the flux is being absorbed. At $x = 0$ some flux remains unabsorbed. The value of dy/dx at $x = 0$ can be found analytically from Eq. (32),

$$\left. \frac{dy}{dx} \right|_{x=0} = \sqrt{2y_\infty - \frac{14}{9}y_\infty^{9/7} - \frac{7}{2}Cy_\infty^{4/7}} \quad (32)$$

This non-zero flux at the base means that the corona is pumping energy into the chromosphere. But the chromospheric gas is not able to radiate it away quickly enough, so it must heat up. This inevitably leads to evaporation of the upper layers of the chromosphere. In order to make a more consistent model, the vertical motion should be taken into account from the start.

6. Model with evaporation

An upwards motion of the gas constitutes an additional vertical advective cooling (MMH). By allowing the constant D to be non-zero, this cooling takes effect. The value of D (being a constant over the entire domain) is determined by adding an additional boundary condition to the system,

$$\left. \frac{dy}{dx} \right|_{x=0} = 0. \quad (33)$$

This yields both a solution for $y(x)$ and a value for D . The evaporation rate is therefore found as an eigenvalue of the system, similar to the case of interstellar cloud evaporation studied by Cowie & McKee (1977) and McKee & Cowie (1977). The solution for the flux dy/dx is plotted in Fig. 3. One can identify three regions in order of decreasing x ,

1. For $x_{eq} \lesssim x < x_{up}$ the corona is nearly in local heating/cooling balance. It is a solution of Eq. (28).

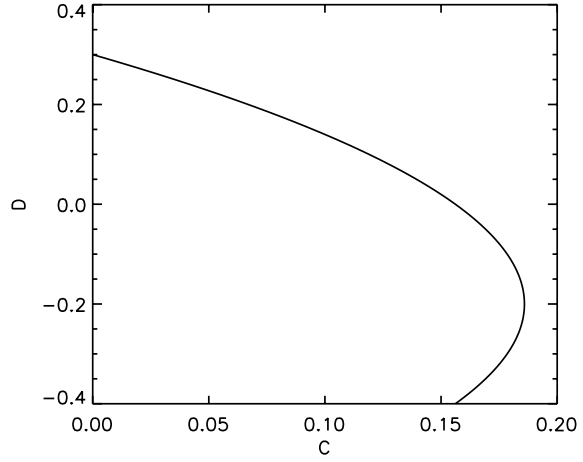


Fig. 4. The relation between the dimensionless evaporation rate D and the coronal accretion rate in the form of C .

2. For $x_{max} < x \lesssim x_{eq}$ the heat flux rises from almost zero to a maximum at x_{max} , as a result of an excess in local heat production.
3. For $0 < x < x_{max}$ the heat flux decreases again to zero as a result of both radiative cooling and evaporation. For this region the solution is approximately a power-law, $T \propto y^{2/7} \propto x^{2/5}$ and the flux goes as $J_c \propto -dy/dx \propto x^{2/5}$.

The value of D , following from these models, depends on the value of C , or in other words on the coronal accretion rate \dot{M}_c . This functional dependence of D on C is plotted in Fig. 4. The curve is closely fitted (to within a few%) by the expression

$$C = \frac{6}{125} \sqrt{15} \left[1 - 4 \left(D + \frac{1}{5} \right)^2 \right] \quad (34)$$

Note that, although the fit is very good, it is not exact. This expression then gives the evaporation rate D as a function of C . There are two branches. The upper branch has evaporation for small C (small \dot{M}_c) and condensation for large C (large \dot{M}_c). For this upper branch the corona (region 1) is in the ADAF mode. The lower branch only has condensation, and the corona is in the SLE mode. Since SLE flows are known to be thermally unstable, this lower branch is not likely to represent any physical situation. The dimensionless evaporation rate for the upper branch is then

$$D = \frac{3}{10} \left[-\frac{2}{3} + \frac{5}{3} \sqrt{1 - \frac{125}{6\sqrt{15}} C} \right] \quad (35)$$

The final formula for the evaporation rate is then (using Eqs. (27) and (18)),

$$\Psi = \frac{3}{10} \sqrt{\frac{21}{2\pi\sqrt{10}}} \frac{\gamma - 1}{\gamma} \sqrt{K_0} \frac{\Omega_K^{5/2} R}{\xi^{3/4} \sqrt{\sigma}} \times f(C/C_{crit}) \sqrt{\dot{M}_c} \quad (36)$$

where $f(C/C_{crit}) \equiv (10/3) D(C)$ is the saturation function, which is unity for $C \ll C_{crit}$, i.e. for $\dot{M}_c \ll \dot{M}_{crit}$.

There is a distinct value of C for which no evaporation nor condensation takes place,

$$C_0 = \frac{21}{25} C_{crit} \quad (37)$$

which corresponds to $\dot{M}_c = (21/25)\dot{M}_{crit}$. For a more dilute corona there will be evaporation, while for a denser corona there will be condensation. It is therefore tempting to conclude that the corona will always tend towards saturation at $\dot{M}_c = (21/25)\dot{M}_{crit}$. But there is a caveat here, which will be discussed below.

Before concluding this section it is interesting to compare the present model with the model of MMH. In the model of MMH part of the evaporated material leaves the system through a transsonic wind, while the other part accretes radially onto the central object. By making a plausible assumption for the radial derivatives of the density and velocity, the evaporation problem is reduced to a 1-D vertical problem. The temperature in the wind is fixed by the conditions at the sonic point, while the pressure at the base, and thereby the evaporation rate, is determined by the balance between wind mass loss and conductive evaporation rate. In this way a well-determined expression for the evaporation rate as a function of radius can be given.

In the present model, a transsonic wind is not considered, although it is not ruled out. The temperature at high z is fixed by the condition that viscous heating is balanced by radial advective cooling and radiative cooling. The evaporation rate is kept a function of the pressure, which is related to the radial accretion rate in the corona. Instead of making any assumption of the radial derivative of density, we have derived a relation between the evaporation rate and the accretion rate in the corona. In the next section we will relate these to the the conductive scale height and to a measure for the effectiveness of evaporation.

7. Efficiency and conductive scale height

The corona can only reach saturation at $\dot{M}_c = (21/25)\dot{M}_{crit}$ when the evaporation is efficient enough. The evaporation rate has to compete with radial ‘mass loss’, i.e. the flow of coronal matter towards the central object. If a corona cannot evaporate the disk efficiently enough, the radial coronal flow will deplete the corona, until a low enough coronal density is reached for evaporation to compete with radial inflow.

To investigate this one should compare the evaporation rate Ψ with the coronal accretion rate \dot{M}_c . Coronal mass conservation in a stationary situation is given by

$$\frac{d\dot{M}_c}{dR} = -2\pi R\Psi \quad (38)$$

Define an effectiveness index χ as $\chi \equiv -d \log \dot{M}_c / d \log R$. One has

$$\chi = 2\pi R^2 \frac{\Psi}{\dot{M}_c} \quad (39)$$

The dimensionless number χ gives the ratio between the evaporation rate and the coronal depletion rate due to radial accretion.

One can also think of it as the ratio between the time scales of vertical motion and radial motion. For $\chi < 0.5$ the evaporation rate is weak and for $\chi > 0.5$ it is strong. It should be kept in mind that for $\chi \geq 0.5$ there exist no self-similar ADAF coronae (Dullemond & Turolla 1998), so that the evaporation models for $\chi > 0.5$ are not self-consistent with respect to angular-momentum conservation. By using Eqs. (39, 36, 30) one obtains

$$\begin{aligned} \chi &= \frac{3}{2} \sqrt{\frac{7}{6\sqrt{15}}} \frac{c\sqrt{K_0 K_1}}{\alpha\sigma} \frac{\gamma-1}{\gamma} \sqrt{\frac{R_g}{R}} \frac{f(\dot{M}_c/\dot{M}_{crit})}{\sqrt{\dot{M}_c/\dot{M}_{crit}}} \\ &\equiv \bar{\chi} f(\dot{M}_c/\dot{M}_{crit}) = 15.1 \frac{\sqrt{\eta}}{\alpha} \sqrt{\frac{R_g}{R}} \frac{f(\dot{M}_c/\dot{M}_{crit})}{\sqrt{\dot{M}_c/\dot{M}_{crit}}} \end{aligned} \quad (40)$$

where $R_g = 2GM/c^2$. In the last step $\gamma = 1.5$.

Closely related to the evaporation efficiency is the conductive scale height z_c . This is the height below which the coronal energy content is significantly drained by thermal conduction. By using Eqs. (21, 30) one can find the dimensionless conductive scale height $\zeta \equiv z_c/H$,

$$\begin{aligned} \zeta &= \frac{10}{21} \sqrt{\frac{7}{3\sqrt{15}}} \frac{c\sqrt{K_0 K_1}}{\alpha\sigma\xi} \sqrt{\frac{R_g}{R}} \frac{x_c}{\sqrt{\dot{M}_c/\dot{M}_{crit}}} \\ &= 7.2 \frac{\sqrt{\eta}}{\alpha} \sqrt{\frac{R_g}{R}} \frac{x_c}{\sqrt{\dot{M}_c/\dot{M}_{crit}}} \end{aligned} \quad (41)$$

The constant x_c is the conductive scale height in the dimensionless x -coordinate, introduced in Sect. 6. As one can infer from the figures, the flux is roughly 20% of its maximum value at $x \simeq 4$, so take the constant $x_c = 4$. For $\zeta \ll 1$ the thermal conduction only affects the lower layers of the corona, while for $\zeta \gtrsim 1$ the conduction affects the entire corona. In fact, if $\zeta \gtrsim 1$ one should expect radial thermal conduction to play an important role.

If $\dot{M}_c \ll \dot{M}_{crit}$, the two dimensionless numbers $\chi = \bar{\chi}$ and ζ are roughly equal. For $\gamma = 1.5$ and $x_c \simeq 4$ one finds

$$\zeta \simeq 1.9\chi \quad (42)$$

One sees that when the evaporation efficiency tends to become strong ($\chi \gtrsim 0.5$), the conductive scale height tends to exceed the size of the system $\zeta \gtrsim 1$. This shows that for strong evaporation the conduction will dominate the entire corona from top to bottom, and radial thermal conduction will be an important effect. So, it is to be expected that for strong evaporation the problem becomes essentially 2-D. A similar conclusion has already been drawn from angular momentum conservation considerations (Abramowicz et al. 1997, Dullemond & Turolla 1998), but in that case a global radial 1-D solution with locally computed evaporation could still not be convincingly excluded. The present conclusion is more dramatic, since it is unlikely that a very non-linear phenomenon like thermal conduction lends itself for dimensional splitting.

Now consider the case when the corona becomes saturated, $\dot{M}_c \rightarrow (21/25)\dot{M}_{crit}$. Since $\dot{M}_{crit} \propto R^{-1/2}$, this saturation

can be only guaranteed when $\chi = \bar{\chi} f(\dot{M}_c/\dot{M}_{crit}) = 0.5$, which implies $\bar{\chi} \geq 0.5$ (since f obeys $f(\dot{M}_c/\dot{M}_{crit}) \leq 1$). This can be achieved for $R \lesssim 10^3 (\eta/\alpha^2) R_G$, but for most radii this means that $\bar{\chi} \gg 1$ and therefore $\zeta \gg 1$. So one should conclude that coronal saturation is necessarily accompanied by strong radial conduction. Under those circumstances the present model breaks down, and fully 2-D models should be used instead.

8. Conclusion

The model presented in this paper described qualitatively the mechanism of disk surface evaporation by thermal conduction. It applies to accretion disc systems around black holes, neutron stars and white dwarfs. Although the model cannot be applied to the case of evaporation very close to a black hole (because of the relativistic velocities of the electrons, and the 2-temperature nature of the plasma) the qualitative picture sketched by this model is applicable to that case as well.

In order for a disk to evaporate completely, the evaporation should be efficient. In a relatively short interval in radius, the complete disk should vanish, and the advection dominated corona should be produced. The corona provides the energy for achieving this, and therefore the corona itself determines how strong the evaporation will be. In an equilibrium situation the evaporation rate should balance the rate of radial inflow of coronal material. If most of the evaporation takes place in a relatively small range in radius, this means that the vertical velocity of the evaporating gas is in the order of the radial accretion velocity of the corona. The model described in this paper shows that in order to achieve this strong evaporation rate, the conductive scale height should be equal or larger than the radius. Consequently, radial thermal conduction will enter the problem. The heat required for evaporation does not come from the upper layers of the corona anymore, but is instead produced closer to the central object and radially transported to the evaporation region. There the flux will be directed down towards the disk surface and evaporate the disk.

In order to model this complete disk evaporation with radial conduction, one should take the radial dimension of the problem into account. One could think of using the usual dimensional splitting procedure to solve the evaporation vertically (using a local ‘heating’ term to account for the divergence in radial heat flux), and the coronal dynamics and radial thermal conduction radially. Unfortunately this procedure is highly questionable because of the very non-linear nature of the formula for conductive heat flux. It seems therefore that one should conclude that disk evaporation by thermal conduction is an essentially 2-D process.

These conclusions do not depend on the micro physical assumptions for evaporation, like the value of the conductivity suppression factor η . The conclusions here can be traced back to energy budget considerations.

If electron thermal conduction is indeed responsible for the evaporation of a Shakura–Sunyaev disk, then there are also important consequences for the spectral modeling of ADAFs. If Comptonized soft-photons from the SSD constitute an important part of the spectrum, then this part emerges from the ADAF region that is strongly affected by radial thermal conduction. It is therefore questionable whether a consistent 2-phase (SSD+ADAF) spectral model can be built without 2-D radiative–hydrodynamic simulations.

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References

- Abramowicz M.A., Chen X., Kato S., Lasota J.-P., Regev O., 1995, *ApJ* 438, L37
- Abramowicz M.A., Igumenshchev I.V., Lasota J.-P., 1997, *MNRAS* 293, 443
- Braginskii S.I., 1963, In: Leontovich M.A. (ed.) *Problems of Plasma Theory*. Atomizdat, Moscow, p. 192
- Cowie L.L., McKee C.F., 1977, *ApJ* 211, 135
- Galeev A.A., Rosner R., Vaiana G.S., 1979, *ApJ* 229, 318
- Hameury J.-M., Lasota J.-P., McClintock J.E., Narayan R., 1997, *ApJ* 489, 234
- Hubeny I., 1989, In: Meyer F., Duschl W.J., Frank J., Meyer-Hofmeister E. (eds.) *Theory of Accretion Disks*. Kluwer Academic Publishers, p. 445
- Ichimaru S., 1977, *ApJ* 214, 840
- Icke V., 1976, *Coronae above accretion discs*. In: Eggleton P., Mitton S., Whelan J. (eds.) *Proc. IAU Symposium 73. Structure and evolution of close binary systems*. Reidel, Dordrecht, p. 267
- Lasota J.-P., 1996, In: Van Paradijs J., Van den Heuvel E.P.J., Kuulkers E. (eds.) *Proc. IAU Coll. 165. Compact Stars in Binaries*. Kluwer Academic Publishers
- Liu B.F., Meyer F., Meyer-Hofmeister E., 1997, *A&A* 328, 247
- McKee C.F., Cowie L.L., 1977, *ApJ* 215, 213
- Meyer F., Meyer-Hofmeister E., 1994, *A&A* 288, 175 (MMH)
- Narayan R., Yi I., 1994, *ApJ* 428, L13
- Narayan R., Yi I., 1995, *ApJ* 452, 710
- Narayan R., Barret D., McClintock J.E., 1997, *ApJ* 482, 448
- Shakura N.I., Sunyaev R.A., 1973, *A&A* 24, 337
- Shapiro S.L., Lightman A.P., Eardly D.M., 1976, *ApJ* 204, 187
- Shaviv G., Wehrse R., 1986, *A&A* 159, L5
- Spitzer L., 1962, *Physics of fully ionized gases*. Interscience Publishers, New York, London, 2nd edition
- Thorne K., Price R.H., 1975, *ApJ* 195, L01
- Tout C., Pringle J., 1992, *MNRAS* 259, 604
- Tschäpe R., Kley W., 1993, *A&A* 273, 169